

HIPPARCHUS ON THE RATIO OF LONGEST DAY TO SHORTEST NIGHT IN EUDOXUS, ARATUS AND ATTALUS

*Part II: The error*¹

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As the title indicates, this is a sequel, or indeed, a correction to a paper, “Hipparchus on the ratio of longest day to shortest night in Eudoxus, Aratus and Attalus (*In Arati et Eudoxi Phaenomena* I.3.10),” which appeared as a contribution to the *Festschrift* to Professor László Török.² That paper argued for a conjecture of Otto Neugebauer, who suggested that in Hipparchus’ report about the two different values for the ratio of longest day to shortest night in two different works of Eudoxus – 5 : 3 in the *Mirror* (Ἐνοπτρον) and 12 : 7 in the *Phaenomena* – the latter value should be emended to 11 : 7.³ In this I argued against Alan Bowen and Bernard Goldstein, who defended the transmitted value.⁴

¹ I am writing this paper in grateful memory of Márta Fehér, the spearhead of teaching and research in history and philosophy of science in Hungary. Márta was a paragon of intellectual honesty and sincerity, an inspiration for all of us who started research and teaching on connected topics. This was matched by her kindness and support – which in my case meant, in a precarious situation, that from 1986 to 1989 my family and I could rent her flat in Buda. Without this help I don’t think I could have concentrated on writing the papers which later made up the brunt of my dissertation, while I was also fulfilling my duties as a TA teaching Greek philosophy at Eötvös University.

² Bodnár 2018.

³ Neugebauer 1975, 733 n. 28.

⁴ Bowen and Goldstein 1991.

Before turning to the issues I intend to tackle in this Part Two, I should set out in some broad outlines the original discussion. Neugebauer had several considerations about the ratio 12 : 7, the value as transmitted in the manuscripts, and about the ratio he suggested, 11 : 7. Interestingly, lack of accuracy of the transmitted value was not among them. Instead, he rested his contention against the ratio 12 : 7 on the grounds that it would imply a division of a circle into 19 equal parts, producing a value for the length of the day that cannot be expressed in unit of hours, and segments of the tropic that cannot even be expressed in degrees.⁵

In response, Bowen and Goldstein argued that the use of equinoctial hours is not attested by the time of Eudoxus.⁶ Furthermore, they claimed that the different traditional values for the ratio of longest day to shortest night are all results of interpolations of the type where between two ratios $a : b$ and $c : d$, a third one, $a + c : b + d$ was inserted.⁷

The most important considerations I could muster against this proposal were that such a

⁵ As further advantages Neugebauer also mentions (i) that the ratio 11 : 7 between longest day and shortest night is attested by Pliny for Athens, in a list of seven arithmetical climata (of System A, on which see Neugebauer 1975, 728–730), or at least, with this emendation the passage in Hipparchus’ *Commentary* would provide “the earliest evidence for the arithmetical climata” – with this very value for Athens in System A, or for Rhodes in System B –, “indeed from prehellenistic times.” (Neugebauer 1975, 733 n. 28).

⁶ The relevant section of their paper, § 3: The earliest occurrence of equinoctial hours and time-degrees in Greek astronomy, summarised their finds in the following fashion “there is no unambiguous evidence in Greek texts [...] for the use of equinoctial hours until P. Hibeh 27, or for the use of seasonal hours until Timocharis [...]”. (Bowen and Goldstein 1991, 240)

⁷ This numerical rule is first employed in Plato’s *Parmenides* (154B1–D3). The sequence leading to 12 : 7 can be set out in the following chart (indicating in square brackets at which step the value was introduced in the series):

1:1 [1]	3:2 [2]	5:3 [3] 12:7 [5] 7:4 [4]	2:1 [1]
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sequence of interpolations would not give an account of which value is assigned to which location, and moreover such a sequence of interpolations would be also problematic, on the grounds that if we start out from the two extreme ratios 1 : 1 and 2 : 1, as Bowen and Goldstein suggest these early astronomers did, any ratio falling between these two extremes can be generated in due course.⁸

This did not automatically mean the acceptance of the value proposed by Neugebauer and the rejection of Bowen and Goldstein's proposal, that we should keep the reading of the manuscripts. Therefore, as a next move I had to assess which value – 12 : 7, the one in the manuscripts, or 11 : 7, as proposed by Neugebauer – makes better sense in the context. The value Eudoxus gives in the *Mirror*, 5 : 3 indicates already some location too far in the North. This is exactly why Hipparchus finds fault with it. It would imply a longest day of 15, and a shortest night of 9 hours, respectively and according to Hipparchus that would be the correct value for regions around the Hellespont, whereas he sets the true value of the longest day for the regions around Greece at 14 3/5 hours.

In the second section of this note I will set out in more detail the context where Hipparchus criticises the value found in Aratus, and in Eudoxus' *Mirror*. At this point we only need to stress that the value transmitted in the manuscripts, the ratio 12 : 7 will be problematic at this juncture, where Hipparchus rebukes Aratus and Attalus that they do not take into consideration this additional value, which features in Eudoxus' other astronomical work, the *Phaenomena*. As a consequence, it would indicate a day longer than 15 hours – so it would point to a region even further to the North. And so, it is not clear what rhetorical role it could play in the passage, what use it could have been for Aratus and Attalus to take

⁸ Neugebauer's proposed value, 11 : 7 can be inserted in the following sequence:

1:1 [1]

3:2 [2] **11:7 [5]** 8:5 [4] 5:3 [3]

2:1 [1]

this different, even more erroneous value on board.

But the value 11 : 7 is also problematic. This is so, because the passage with 11 : 7 reads

One would be even more astonished how on Earth he [Attalus] did not realise that Eudoxus sets out the issue differently in his other work and writes that the section of the tropic above the horizon is in a ratio of 11 to 7 to the section below the Earth, and the people around Philip and many others have recorded similarly, with the exception that they have arranged the risings and settings of the stars according to the region around Greece, but they are in error with respect to the inclination of this region. (I.3.10)⁹

If we replace the ratio 12 : 7 with the one proposed by Neugebauer, as I did here, we reach a double-bind.

[This emended ratio] is *too* close to the true ratio given by Hipparchus. Hipparchus establishes that the longest day is 14 and 3/5 hours (14 hours and 36 minutes), whereas the emended ratio, 11 : 7, gives a longest day of 14 and 2/3 hours (14 hours and 40 minutes). The difference is just 4 minutes – or if one were to speak about the segments of the circle of the summer tropic, as Aratus and Eudoxus formulate their claim, the difference is altogether one degree, 1/360 part of the whole circle. With this minute difference one needs to raise the question not only whether it is fair on Hipparchus' part, but whether it is at all credible that he should rebuke Philip, and many other astronomers that although on the whole

⁹ ἔτι δὲ μᾶλλον θαυμάσειεν ἂν τις, πῶς ποτε οὐκ ἐπέστησε τοῦ Εὐδόξου ἐν τῷ ἐτέρῳ συντάγματι διαφόρως ἐκθεμένου καὶ γράφοντος, ὅτι τὸ ὑπὲρ γῆν τοῦ τροπικοῦ τμήμα πρὸς τὸ ὑπὸ γῆν λόγον ἔχει, ὃν <ἔχει> τὰ ια' πρὸς τὰ ζ', ὁμοίως δὲ τούτῳ καὶ τῶν περὶ Φίλιππον ἀναγραφόντων καὶ ἄλλων πλείονων, πλὴν ὅτι συντετάχασιν μὲν τὰς συνανατολάς τε καὶ συγκαταδύσεις τῶν ἄστρον ὡς ἐν τοῖς περὶ τὴν Ἑλλάδα τόποις τετηρημένων, κατὰ δὲ τὸ ἔγκλημα τῶν τόπων τούτων διημαρτήκασιν. (Manitius' text with Neugebauer's emendation)

their account is in accordance with celestial phenomena as observed from Greece, they nevertheless are in error, because their division of the tropical circle into segments corresponding to longest day and shortest night is one degree off the mark. (Bodnár 2018, p. 693)¹⁰

As a result, in a final section of the paper, I had to investigate what degree of precision is usually expected by Hipparchus, what the magnitude of error is that he most likely would censure. I was aware that different kinds of observations could be assigned different margins of admissible error. Hence, after going through a few different cases, I turned to assess the contrast between the first, commentary part of the work – where Hipparchus set the length of the longest day in Greece to $14 \frac{3}{5}$ hours –, and the second, expository part, where this value is set to $14 \frac{1}{2}$ hours. One way of reading this is take it as an indication of tolerance of an error of 6 minutes, which translates to $1 \frac{1}{2}$ degrees on the tropical circle. However, I took the opposite path, suggesting that the former value was meant for mainland Greece, whereas the second one is introduced for Rhodes, which is slightly to the South. The fact that the two values might be introduced for two different regions could mean that the tolerance of error can be even smaller than the difference between them.

I used this supposition then as a stepping stone, to conclude

¹⁰ I added at this point in note 24 that “It can also be illuminating to take into account that as Lloyd: [1982,] pp. 143–144, stresses “ancient astronomers could tell the time at night to an accuracy of within ten minutes, which will correspond to between two and three degrees in the motion of the stars on the celestial equator. In line with this, no actual recorded observation in Ptolemy is more precise than to within one-sixth of an hour.” Accordingly, the differences of these lengths of the longest days (or of the shortest nights) are all computed – or otherwise derived – values which could not be ascertained by direct observation. (“Otherwise derived”: in principle, such details could be read off from suitably constructed representations or diagrams. Cf. Neugebauer 1975, p. 279, who remarks about Hipparchus’ way of determining the positions of the stars in the second part of the *Commentary* that it “is convenient both for readings on a globe and for graphic construction or plane trigonometric computation based on stereographic projection [...], assuming that the latter was known to Hipparchus.”)

This, then, allows that just as he makes a clear and systematic distinction between the phenomena as observed from Rhodes and from Athens, he can have a similar reason to distinguish between a place where the longest day is 14 hours and 36 minutes long, and another one where the longest day is 4 minutes longer. Otherwise put: Hipparchus may find it vindicated to charge astronomers with an error if they use this latter value [as proposed by Neugebauer] in place of the former. (Bodnár 2018, p. 697)

All in all, this way I could argue that Hipparchus had every reason to mention the way better value of Eudoxus' *Phaenomena*, while rebuking Aratus' and Attalus' use of the value, taken from the *Mirror*, which is even wider of the mark, and at the same time indicating that the value of the *Phaenomena*, albeit preferable, is still problematic.

There are several presuppositions along which one could query this argument. Most importantly, note that the last, stepping stone move is inherently problematic: It is pretty much like a step in a *sōritēs* argument. One may have every reason to grant it, and also to resist it. My inclination to accept this move undoubtedly came from the desire to reach a position in which Hipparchus can accomplish the feat of killing two birds with a single stroke.

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Before turning to the actual topic of this sequel and setting out how this impasse can be overcome, we need to have a look at the broader context of Hipparchus' critical remarks, and what he claims has been achieved by the end of Chapter 3 of Book I of the *Commentary*.

In the first two chapters Hipparchus substantiates his claim that Aratus and Attalus follow very closely Eudoxus' *Phaenomena*. This is just fine as far as Aratus is concerned – he writes poetry. Attalus, however, does not get off the hook: he is an astronomer, and wrote

an exposition of Aratus' *Phaenomena*, where he claimed that the contents of the poem correspond to astronomical reality. Accordingly, Hipparchus by tying the astronomical details of the poem to Eudoxus' *Phaenomena* touches a nerve: although there is nothing much wrong as far as Aratus is concerned, Attalus, however, should have known that progress in astronomy made much of Eudoxus' claims a thing of the past. In setting out Aratus' *Phaenomena* he should have called systematically attention to the errors and discrepancies involved in the poem – much like what Hipparchus will do from Chapter 4 on. Chapter 3 has the special status in this regard, because – as Hipparchus points out – in this instance Aratus (and following him, Attalus) rest on what Eudoxus writes in the *Mirror*, and do not take the value of the *Phaenomena* into consideration.

After calling attention to this, and after mentioning that Philip and his circle was in agreement with the value as set out in Eudoxus' *Phaenomena*, Hipparchus concludes the chapter with the following programmatic statement:

Hence leaving this error aside, we have investigated the whole of their composition as against the horizon in Greece. For it is not appropriate for someone aiming at truth, but rather for someone zealous about frivolities, to attack them in each and every instance that is in contradiction with their discredited assumption, even when these are said in accordance with what is seen in Greece in the sky. (I.3.11)¹¹

In other words, the following chapters, from Chapter 4 on, will assess the contents of Aratus' *Phaenomena*, and Attalus' explanatory remarks to this work only against the actual celestial phenomena as observed from Greece. They will not come back, again and again

¹¹ Παραπέμψαντες <οὖν> τοῦτο τὸ ἀγνόημα τὴν ὅλην αὐτῶν σύνταξιν ἐπεσκεψάμεθα πρὸς τὸν ἐν τῇ Ἑλλάδι ὀρίζοντα. οὐδὲ γὰρ φιλαλήθους, ἀλλὰ κενοσπούδου, τὸ κατὰ πάντα μαχόμενον τῇ διεψευσμένη ὑποθέσει ἐπιλαμβάνεσθαι αὐτῶν, κἂν τύχη συμφώνως λεγόμενα τοῖς ἐν τῇ Ἑλλάδι φαινομένοις.

to the additional critical quip of how much more erroneous these records would come out if one were to assess them as against the phenomena in the region lying further to the North, where the ratio of longest day and shortest night is 5 : 3.¹²

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Now we can turn to what makes a fundamental change about the assessment of the import of the passage about the ratio of longest day to shortest night, the reading of the text proposed by Alexander Jones.¹³

Jones rejects Manitius' conjecture at p. 28, 14, πλὴν ὅτι, and suggests that the manuscripts' πλὴν ἐπὶ συντετάχασι should be corrected as resulting from itacism, according to which – among several other vowels and diphthongs – the diphthong ει was also pronounced as i. Especially in case of dictation such errors could creep in easily. Moreover, the articulation of the passage proposed by Jones makes the addition of οὖν in p. 28, 19¹⁴ superfluous:

¹² See here For a case when Hipparchus nevertheless uses an argument of the second kind, see I.7.19–22: Here Aratus (and Attalus) are censured for Aratus' assertion that the constellation Cepheus rises and sets from the head all the way to its belt. Criticising this statement, Hipparchus sets out that from Greece only a smaller part – from head to shoulders – rises and sets, the rest is circumpolar (I.7.19–21). Then he continues with the remark that “the error turns out to be much greater if we assumed the inclination of the world he himself proposes: for where the longest day is 15 hours, there the whole of the constellation Cepheus is circumpolar” (πολλῷ δὲ μείζον γίνεται τὸ ἀγνόημα, κἄν ὑποθώμεθα τὸ καθ' ἑαυτὸ <ν> ἔγκλημα τοῦ κόσμου· ὅπου γὰρ ἡ μεγίστη ἡμέρα ὠρῶν ἐστι ιε', ἐκεῖ ὅλος ὁ Κηφεὺς ἐν τῷ ἀρκτικῷ φέρεται. I.7.22, where I read καθ' ἑαυτὸν instead of καθ' ἑαυτὸ of family A of the manuscripts, along the lines of the emendation by Scaliger, who reads τὸ κατ' αὐτὸν ἔγκλημα). Note, however, that this is not in violation of the limits of criticism as announced in I.3.11. It is only after arguing that the assertion of Aratus is not borne out by what can be seen in the sky from Greece that Hipparchus launches this further, fully devastating objection.

¹³ Professor Jones showed me his unpublished note (“Hipparchus *In Arat. et Eudox.* 1.3.5-12, ed. Manitius 26-28,” now available at <https://archive.nyu.edu/handle/2451/63346>) when I sent him my paper and allowed me to refer to his note in this piece.

¹⁴ Already Dionysius Petavius (Denise Pétau) indicated in his edition that there was something amiss

Once the clause beginning with Πλήν ἔπει is detached from the report about the people about Philip (and many others), it leads over seamlessly to what was printed as the beginning of the following section, section 11, and as a result the clause beginning section 11 – which is no longer the beginning of a self-standing sentence – does not need such a particle for connecting it up with the context.

With these changes the text reads:

One would be even more astonished how on Earth he [Attalus] did not realise that Eudoxus sets out the issue differently in his other work and writes that the section of the tropic above the horizon is in a ratio of 11 to 7 (with Neugebauer’s emendation, or 12 to 7 with the manuscripts) to the section below the Earth, and the people around Philip and many others have recorded similarly.

Since, however, **they** arrange the risings and settings of the stars according to the region around Greece, although **they** are in error with respect to the inclination of this region, leaving this error aside, we have investigated the whole of **their** composition as against the horizon in Greece. For it is not appropriate for someone aiming at truth, but rather for someone zealous about frivolities, to attack them in each and every instance that is in contradiction with **their** discredited assumption, even when these are said in accordance with what is seen in Greece in the sky. (I.3.10–11, with emphases introduced by me in bold)¹⁵

between what is the end of section 10 and the beginning of section 11, and tentatively suggested that ἡμεῖς οὖν could be added before παραπέμψαντες. (Petavius’ edition was available to me in his *Opus de doctrina temporum, in quo Uranologium etc.*, Antwerp: Georgius Gallet, 1703, the passage is on p. 101 in that edition. This edition is also available in the Internet Archive, at https://archive.org/details/bub_gb_-c9KaFvmqLgC checked on 28 August 2021.)

¹⁵ ἔτι δὲ μᾶλλον θαυμάσειεν ἂν τις, πῶς ποτε οὐκ ἐπέστησε τοῦ Εὐδόξου ἐν τῷ ἐτέρῳ συντάγματι διαφόρως ἐκθεμένου καὶ γράφοντος, ὅτι τὸ ὑπὲρ γῆν τοῦ τροπικοῦ τμήμα πρὸς τὸ ὑπὸ γῆν λόγον ἔχει, ὃν <ἔχει> τὰ ἰα’

It is clear that the correction, and the ensuing rearrangement of the text accomplishes that the error mentioned and tacked on to the end of section 10 on Pétau's and Manitius's articulation of the text, is no longer imputed to the people who opt for the alternative proffered by Eudoxus in his *Phaenomena*. Instead, after mentioning this value with the roll call of the Philippons and many others, Hipparchus returns to Eudoxus', Attalus' and Aratus' error, the one he started out chastising from section 5 on. Otherwise put, the error these lines return to for a last time is not one committed by the Philippons, nor by Eudoxus in his *Phaenomena*. Hence there is no need to establish what degree of precision Hipparchus should expect with which he can cut both ways: first, in a rebuke against Attalus, that he was sticking to the erroneous 5 : 3 ratio instead of this one, but also in the very next clause pointing out that it is still unacceptable as the true value for the ratio of the longest day and the shortest night. In the corrected text this latter charge of ignorance does not apply to the alternative value introduced in the previous lines. Instead, the rebuke remains to be targeted at the original lot, who could not establish the inclination, that is the latitude of the regions around Greece.

Jones' correction of the text is clearly preferable to Manitius' conjectures on purely philological grounds. Some final assessment of the narrative structure of the two options is still in order. On Jones's correction the line of thought is somewhat odd: Chapter 3, after summarising in sections 1–4 the results of the first two chapters about the dependence of Aratus and Attalus on Eudoxus' *Phaenomena*, in sections 5–9 turns to setting out this

πρὸς τὰ ζ', ὁμοίως δὲ τούτῳ καὶ τῶν περὶ Φίλιππον ἀναγραφόντων καὶ ἄλλων πλειόνων.

Πλὴν ἔπει συντετάχασιν μὲν τὰς συνανατολάς τε καὶ συγκαταδύσεις τῶν ἄστρον ὡς ἐν τοῖς περὶ τὴν Ἑλλάδα τόποις τετηρημένων, κατὰ δὲ τὸ ἔγκλημα τῶν τόπων τούτων διημαρτήκασιν, παραπέμψαντες τοῦτο τὸ ἀγνόημα τὴν ὅλην αὐτῶν σύνταξιν ἐπεσκεψάμεθα πρὸς τὸν ἐν τῇ Ἑλλάδι ὀρίζοντα. οὐδὲ γὰρ φιλαλήθους, ἀλλὰ κενοσπούδου, τὸ κατὰ πάντα μαχόμενον τῇ διεψευσμένη ὑποθέσει ἐπιλαμβάνεσθαι αὐτῶν, κἂν τύχη συμφώνως λεγόμενα τοῖς ἐν τῇ Ἑλλάδι φαινομένοις.

fundamental error of Aratus and Attalus, following in this case Eudoxus' *Mirror*. After these five sections, the sentence about Eudoxus' *Phaenomena* introduces yet further protagonists on the scene: the people around Philip and many others. It is only after this, that Hipparchus at p. 28, 14, without explicitly signalling, returns to speaking about the error censured in sections 5–9, and closes the chapter in the last two sections with the programmatic statement quoted above about the remaining chapters of Book I.

Granted, the switch back to the original targets of criticism after what one could take as a parenthetical remark about the other value is abrupt. But this difficulty is just as well present in the line of argumentation of the text with Manitius' conjectures. In this version the criticism formulated at p. 28, 14–18 is levelled against all these authorities. That requires that by Hipparchus' lights the value attributed to them cannot be acceptable. But then starting from p. 28, 19 the text with the programmatic statement about Hipparchus' procedures in what lies ahead has to return to the original targets of criticism: Hipparchus' commentary remains restricted to the original culprits, Aratus and Attalus, and behind them Eudoxus. In this case, however, the difficulty one could find with the corrected text applies at least as much to the version with Manitius' conjectures.

Or even more. The passage on the articulation of the corrected text has a continuous and connected thread of third person plural personal pronouns referring to the same people after the parenthetical remark. Note, however, that contrasted to this on the articulation of text with Manitius' conjectures the similarly abrupt change happens after the two first occurrences of the pronoun 'they'. These refer to the larger group, containing also the Philipians and others. Then, the next sentence, still about 'them' refers to a different, smaller group of people – making this change even more abrupt.

In closing three final points can be made. First, the force of Jones' correction should be clearly articulated. Most importantly the difference is not so much in the meaning of *πλήν*.

In all three alternatives – the text as it stands in the manuscripts, then the one with Manitius' conjectures, and then the text with Jones' correction – the meaning of this conjunction is adversative. In the case of the text of the manuscripts and with Manitius' conjecture the clause introduced by *πλήν* will be attached to what went before. It is the presence of the subordinate conjunction *ἐπεὶ* after the adversative conjunction *πλήν* in Jones' correction that makes the crucial difference. These two words together (“since, however”) see to it that the clause introduced by them is linked prospectively to what will be announced in the following lines, and that the line of argumentation returns to the discussion of Aratus and Attalus at this point.

As a consequence, one may venture that the value attributed to Eudoxus' *Phaenomena*, and to the other authorities in the manuscripts may well have crept into the text concurrently with the mistake in the manuscript tradition which disfigured the subordinate conjunction *ἐπεὶ* (“since”) into the preposition *ἐπί* or the prefix *ἐπι-*. Once the clause lost its conjunction that connected it to what comes afterwards in the following clauses, the remark speaking about a mistake on these people's part was understood as applying to those mentioned in the immediately preceding clause. A value that could be admissible by Hipparchus' (or by his editors' and copyists') lights could not possibly feature in the previous lines then. Accordingly, Jones' correction apart from resulting in a cleaner text also gives an account how the mistaken ratio 12 : 7 could crop up in the text at this point.

And finally, I also have to admit what should have become obvious by now. A key advantage of the text with Jones' correction is that it makes redundant the somewhat contorted argumentation in the earlier paper, which through the addition of this Part II has now become Part I of the sequence. A simpler and much more straightforward argument is possible, one that easily integrates Neugebauer's conjecture for the ratio of longest day and

shortest night.¹⁶

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¹⁶ I would like to thank the anonymous referee for an important suggestion about the presentation of the issues tackled in Part I.

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